hw set 12
sect 5.10
p 431

6. $\int_{-\infty}^{0} \frac{1}{2x-5}\,dx$

$= \lim_{t\to-\infty} \frac{1}{2}\int_t^0 \frac{2}{2x-5}\,dx$

$= \lim_{t\to-\infty} \frac{1}{2}\ln|2x-5|\Big|_t^0$

$= \lim_{t\to-\infty} \frac{1}{2}\ln 5 - \frac{1}{2}\ln|2t-5| = -\infty$

divergent

8. $\int_0^{\infty} \frac{x}{(x^2+2)^2}\,dx$

$= \lim_{t\to\infty} \int_0^t \frac{x}{(x^2+2)^2}\,dx$

$u = x^2+2$
$du = 2x\,dx$
$\int u^{-2}\,du = -u^{-1}$

$= \lim_{t\to\infty} \frac{1}{2}\int_0^t \frac{2x}{(x^2+2)^2}\,dx$

$= \lim_{t\to\infty} \frac{1}{2} \frac{-1}{x^2+2}\Big|_0^t$

$= \lim_{t\to\infty} -\frac{1}{2}\left(\frac{1}{t^2+2} - \frac{1}{2}\right) = \frac{1}{4}$ convergent

14. $\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx$

$u = +x^3$
$du = +3x^2\,dx$
$+\frac{1}{3}du = x^2\,dx$

$= \int_{-\infty}^{\infty} +\frac{1}{3} e^{-u}\,du$

$= \lim_{t\to\infty} \int_0^t \frac{1}{3}e^{-u}\,du$

$+ \lim_{t\to-\infty} \int_t^0 \frac{1}{3}e^{-u}\,du$

$\lim \left[-\frac{1}{3}e^{-u}\Big|_0^t + -\frac{1}{3}e^{-u}\Big|_t^0\right]$

$= \lim_{t\to\infty}\left[-\frac{1}{3}e^{-t} + \frac{1}{3}\right] + \lim_{t\to-\infty}\left[-\frac{1}{3} + \frac{1}{3}e^{-t}\right]$

$= \infty$ divergent

15. $\int_0^{\infty} s e^{-5s}\,ds$

$= \lim_{t\to\infty} \int_0^t s e^{-5s}\,ds$ use table #96

$= \lim_{t\to\infty} \left[\frac{1}{25}(-5s-1)e^{-5s}\Big|_0^t\right]$

$= \lim_{t\to\infty} \left[\frac{1}{25}[-5t-1]e^{-5t} + \frac{1}{25}\right]$

$= \lim_{t\to\infty} \left[-\frac{1}{5}te^{-5t} - \frac{1}{25}e^{-5t} + \frac{1}{25}\right]$

$= -\frac{1}{5}\left(\lim_{t\to\infty} te^{-5t}\right) + \frac{1}{25}$

$= -\frac{1}{5}\left[\lim_{t\to\infty} \frac{t}{e^{5t}}\right] + \frac{1}{25} = -\frac{1}{5}\left[\lim_{t\to\infty} \frac{1}{5e^{5t}}\right] + \frac{1}{25} = \frac{1}{25}$

21. $\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx = \int_{-\infty}^{0} \frac{x^2}{9+x^6}\,dx + \int_0^{\infty} \frac{x^2}{9+x^6}\,dx$

$\lim_{t\to-\infty} \int_t^0 \frac{x^2}{9+(x^3)^2}\,dx$

$= \lim_{t\to-\infty} \frac{1}{3}\int_t^0 \frac{3x^2}{9+(x^3)^2}\,dx$

$= \lim_{t\to-\infty} \frac{1}{3}\cdot\frac{1}{3}\tan^{-1}\left(\frac{x^3}{3}\right)\Big|_t^0$

$= \lim_{t\to-\infty} -\frac{1}{9}\tan^{-1}\left(\frac{t^3}{3}\right) = -\frac{1}{9}\left(-\frac{\pi}{2}\right)$

$\lim_{t\to\infty} \int_0^t \frac{x^2}{9+x^6}\,dx = \lim_{t\to\infty} \frac{1}{9}\tan^{-1}\left(\frac{x^3}{3}\right)\Big|_0^t$

$= \lim_{t\to\infty} \frac{1}{9}\tan^{-1}\left(\frac{t^3}{3}\right) = \frac{1}{9}\left(\frac{\pi}{2}\right)$

$\therefore \int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx = \frac{\pi}{9}$

convergent

22. $\int_0^{\infty} \frac{e^x}{e^{2x}+3}\,dx$

$\int \frac{e^x}{e^{2x}+3}\,dx$ $u = e^x$, $du = e^x\,du$

$= \int \frac{1}{u^2+3}\,du = \int \frac{1}{u^2+(\sqrt{3})^2}\,du$

$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{e^x}{\sqrt{3}}\right)$

So $\lim_{t\to\infty} \int_0^t \frac{e^x}{e^{2x}+3}\,dx$

$= \lim_{t\to\infty} \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{e^x}{\sqrt{3}}\right)\Big|_0^t$

$= \lim_{t\to\infty} \left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{e^t}{\sqrt{3}}\right) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

$= \frac{1}{\sqrt{3}}\left[\lim_{t\to\infty} \tan^{-1}\left(\frac{e^t}{\sqrt{3}}\right)\right] - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{1}{\sqrt{3}}\cdot\frac{\pi}{2} - \frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}$

$= \frac{\pi\sqrt{3}}{9} = \frac{\pi}{3\sqrt{3}}$

29. $\int_{-1}^{1} \frac{e^x}{e^x-1}\,dx = \int_{-1}^{0} \frac{e^x}{e^x-1}\,dx + \int_{0}^{1} \frac{e^x}{e^x-1}\,dx$

$\int_0^1 \frac{e^x}{e^x-1}\,dx$ $\quad u = e^x - 1$, $du = e^x\,dx$ $\Rightarrow \int \frac{1}{u} = \ln u$

$= \ln|e^x - 1| \Big|_0^1$ at $x=0$ trouble

$= \ln(e-1) - \lim_{t\to 0} \ln(e^t - 1)$

$= \ln(e-1) - (-\infty) = \infty$

divergent

31. $\int_0^2 z^2 \ln z\,dz$

$\int x^2 \ln x\,dx$

$u = \ln x \quad du = \frac{1}{x}$

$dv = x^2 \quad v = \frac{x^3}{3}$

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$

$= \lim_{t\to 0} \int_t^2 z \ln z\,dz$

$= \lim_{t\to 0} \left[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \Big|_t^2\right]$

$= \lim_{t\to 0} \left[\frac{8}{3}\ln 2 - \frac{8}{9} - \frac{1}{3}t^3 \ln t + \frac{1}{9}t^3\right]$

$= \frac{8}{3}\ln 2 - \frac{8}{9} + \lim_{t\to 0} \left[\frac{1}{3}t^3\left(\frac{1}{9} - \ln t\right)\right]$

now, $\lim_{t\to 0} \frac{1}{3}t^3\left(\frac{1}{9} - \ln t\right) \rightarrow 0\cdot\infty$

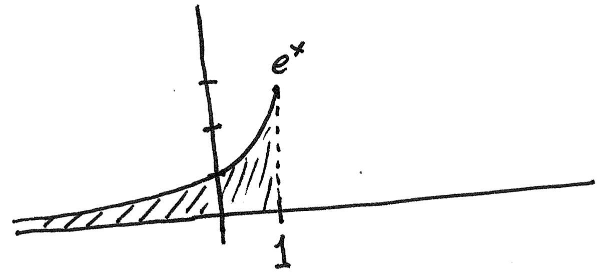
$= \lim_{t\to 0} \frac{\left(\frac{1}{9} - \ln t\right)}{3t^{-3}}$

$= \lim_{t\to 0} \frac{-1/t}{-9t^{-4}}$

$= \lim_{t\to 0} \frac{1}{9}t^3 = 0$

$\therefore \int_0^2 z \ln z\,dz = \frac{8}{3}\ln 2 - \frac{8}{9}$

33. $S = \{(x,y) \mid x \le 1,\ 0 \le y \le e^x\}$



Area $= \int_{-\infty}^{1} e^x\,dx$

$= \lim_{t\to -\infty} \int_t^1 e^x\,dx$

$= \lim_{t\to -\infty} e^x \Big|_t^1$

$= \lim_{t\to -\infty} e - e^t = e$

41. $\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx$

$0 \le \cos^2 x \le 1$

$\int_1^\infty \frac{\cos^2 x}{1+x^2}\,dx \le \int_1^\infty \frac{1}{1+x^2}\,dx$

$\int_1^\infty \frac{1}{1+x^2}\,dx$

$= \lim_{t\to\infty} \int_1^t \frac{1}{1+x^2}\,dx$

$= \lim_{t\to\infty} \tan^{-1}(x) \Big|_1^t$

$= \lim_{t\to\infty} \tan^{-1}(t) - \tan^{-1}(1)$

$= \pi/2 - \arctan(1)$

$= \pi/2 - \pi/4$

$= \pi/4$